

# On Stability of Flat Band Modes in a Rhombic Nonlinear Optical Waveguide Array

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The quasi-one-dimensional rhombic array of the waveguides is considered. In the nonlinear case the system of equations describing coupled waves in the waveguides has the solutions that represent the superposition of the flat band modes. The property of stability of these solutions is considered. It was found that the flat band solution is unstable until the power threshold be attained.

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## I. INTRODUCTION

The optical simulations of the different phenomena of the condensed matter physics [1–4], the quantum physics [5, 6], and the cosmology [7–10] are the object of resent investigation. Under consideration of the 2D electron systems it was found that the presence of the third atom in the unit cell of the lattice leads to emerging of a flat band between conventional energy zones. Similar optical lattices can be realized by means of waveguides as nodes of the lattice [11–13]. Some kinds of the optical lattices that demonstrate the photonic spectrum with flat band have been discussed in [14–16]. If the electric field in the optical lattice is made up from the flat band modes then this field demonstrates the diffractionless propagation along waveguide array.

Recently the quasi-one-dimensional array of the waveguides containing the three linear chain of waveguides was considered [14]. The central chain marked as A-type chain is placed between two chains of waveguides, which are marked as B- and C- type ones. These chains of waveguides are shifted relative to the A-chain at one half of period of the lattice. Interaction between waveguides is due to tunnel coupling. Furthermore, coupling between A-B and A-C waveguides takes place only. This waveguide system alike to a double zigzag array or to an array of rhombus (see Fig. 1).

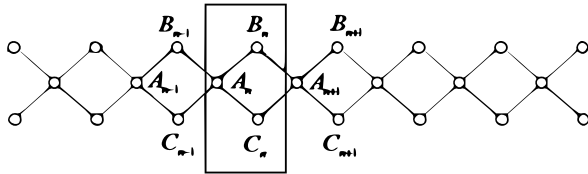


FIG. 1: The rhombic array of waveguides. The unit cell is shown by rectangular box.

This waveguide array named as the quasi-one-dimensional rhombic array has been studied in [14, 17, 18] in the case of linear optical waveguides.

The purpose of this paper is to study the stability of the electromagnetic field distribution in the quasi-one-dimensional rhombic array of the nonlinear waveguides. The nonlinearity is described by the susceptibility of third order. In Sec. III the some nonlinear analogies of the superposition of the flat band modes are found. The discrete diffraction for these electric field distributions over the waveguides is absent. The stability of these solutions is investigated by the use of the linear stability analysis in Sec. IV. Taking into account that the field distribution over the waveguide is the superposition of the band modes, we can make inferences about stability band mode from analysis of the stability properties of the field distribution.

## II. MODEL AND BASIC EQUATIONS

It is assumed that all waveguides are manufactured from a nonlinear dielectric, which is characterized by Kerr nonlinearity. System of equations describing coupled waves in this quasi-one-dimensional rhombic nonlinear optics waveguide array (RNOWA) has the following form

$$\begin{aligned} i \left( \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \zeta} \right) A_n &= c_1 (B_n + B_{n-1}) + \\ &\quad + c_2 (C_n + C_{n-1}) + \mu_1 |A_n|^2 A_n, \\ i \left( \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \zeta} \right) B_n &= c_1 (A_{n+1} + A_n) + \\ &\quad + \mu_2 |B_n|^2 B_n, \\ i \left( \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \zeta} \right) C_n &= c_2 (A_{n+1} + A_n) + \mu_3 |C_n|^2 C_n. \end{aligned} \quad (1)$$

Here  $A_n$ ,  $B_n$  and  $C_n$  are dimensionless slowly varying amplitudes of the electric fields propagating in waveguides of RNOWA. The sub-indices are numbering unit cells (see Fig.1). It is assumed that the phase matching condition is satisfied. The coefficients  $c_1$  and  $c_2$  specify the coupling between waveguides from different chains. The parameters  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  represent the

self-interaction effect in waveguides. If  $c_1 = c_2$  and  $\mu_1 = \mu_2 = \mu_3 = 0$ , the system of equations (1) is reduced to the system of the linear equations considered in [17, 18]. The symmetric rhombic array, where  $c_1 = c_2 = 1$  and  $\mu_b = \mu_c$  will be considered. This contraction allows us to reduce the system of equations (1) to following equations

$$\begin{aligned} i \frac{\partial}{\partial \zeta} A_n &= (B_n + B_{n-1}) + (C_n + C_{n-1}) + \mu_1 |A_n|^2 A_n, \\ i \frac{\partial}{\partial \zeta} B_n &= (A_{n+1} + A_n) + \mu_2 |B_n|^2 B_n, \\ i \frac{\partial}{\partial \zeta} C_n &= (A_{n+1} + A_n) + \mu_2 |C_n|^2 C_n. \end{aligned} \quad (2)$$

If the power per one unit cell is defined by expression  $W_n = |A_n|^2 + |B_n|^2 + |C_n|^2$ , the equation

$$\frac{\partial W_n}{\partial \zeta} + i [A_{n+1}(B_n^* + B_n^*) + A_n^*(B_{n-1} + C_{n-1}) - c.c.] = 0.$$

can be obtained from the system of equations (2). The local value

$$f_n = i(A_n D_{n-1}^* - A_n^* D_{n-1}), \quad (3)$$

is introduced, where  $D_n = B_n + C_n$ . So, the equation for  $W_n$  can be rewritten now as

$$\frac{\partial W_n}{\partial \zeta} + (f_{n+1} - f_n) = 0. \quad (4)$$

The expression in brackets can be interpreted as the discrete divergence of the current density  $f_n$  in 1D space. The equation (4) is the discrete continuity equation.

There are three constrains: (a)  $A_n = 0$ ,  $B_n = -C_n$ , (b)  $A_n = (-1)^n A$ ,  $B_n = (-1)^n B$ ,  $C_n = (-1)^n C$ , and (c)  $A_n = (-1)^n A$ ,  $B_n = (-1)^n B$ ,  $C_n = (-1)^{n+1} C$ . For all these constrains the (4) is hold. However, in the case of (a) and (c) the current density  $f_n$  is zero for any  $A$ ,  $B$  and  $C$ . In the case of (b)  $f_n = (-i)(AD^* - A^*D)$ . This current will be zero only if  $A = 0$ . In the linear case the distribution of the kind (a) corresponds to the flat band modes of [14, 17, 18]. In the RNOWA the electric field distributions under the constrains (a) and (c) can be considered as a nonlinear version of the superposition of the flat band modes. In following the electric field distribution (a) will be studied.

### III. THE FLAT BAND SOLUTIONS

With taking into account constrain  $A_n = 0$ ,  $B_n = -C_n$ , the system of equations (2) can be represented as

$$\begin{aligned} i \frac{\partial}{\partial \zeta} A_n &= 0, \quad i \frac{\partial}{\partial \zeta} B_n = \mu_2 |B_n|^2 B_n, \\ i \frac{\partial}{\partial \zeta} C_n &= \mu_2 |C_n|^2 C_n. \end{aligned} \quad (5)$$

Defining the real variables  $a_n$ ,  $b_n$ ,  $c_n$ ,  $\varphi_a$ ,  $\varphi_b$  and  $\varphi_c$  from the formulas  $A_n = a_n \exp(i\varphi_a)$ ,  $B_n = b_n \exp(i\varphi_b)$  and  $C_n = c_n \exp(i\varphi_c)$ , the real equations can be derived

$$\frac{\partial a_n}{\partial \zeta} = 0, \quad \frac{\partial b_n}{\partial \zeta} = 0, \quad \frac{\partial c_n}{\partial \zeta} = 0,$$

$$\frac{\partial \varphi_{an}}{\partial \zeta} = -\mu_1 a_n^2, \quad \frac{\partial \varphi_{bn}}{\partial \zeta} = -\mu_2 b_n^2, \quad \frac{\partial \varphi_{cn}}{\partial \zeta} = -\mu_2 c_n^2.$$

As the case of  $A_n = 0$  is considered the phase  $\varphi_{an}$  is indeterminate. The amplitudes  $b_n$  and  $c_n$  are constant,  $b_{n0}$  and  $c_{n0} = -b_{n0}$ .

With taking into account this result the solutions of these equations can be written as

$$\varphi_b = \varphi_c = -\mu_2 b_{n0}^2 \zeta.$$

The initial phases are constants of integration and can be chosen to be zero. Thus the solution of the the system of equations (5) reads as

$$\tilde{A}_n = 0, \quad \tilde{B}_n = b_{n0} e^{-i\mu_2 b_{n0}^2 \zeta}, \quad \tilde{C}_n = -b_{n0} e^{-i\mu_2 b_{n0}^2 \zeta}. \quad (6)$$

The electric field distribution (6) characterized by the diffractionless propagation along waveguides will referred to as flat band solution of the equation (2).

### IV. STABILITY OF THE FLAT BAND SOLUTION

Let us consider the homogeneous distribution of the electric field amplitudes over waveguide array  $b_{n0} = b_0$ . The stability of the solution (6) can be analyzed by introducing small perturbations into the electric field amplitudes:

$$\begin{aligned} A_n &= p_n e^{-i\mu_2 b_0^2 \zeta}, \\ B_n &= \tilde{B}_n + b_n = (b_0 + q_n) e^{-i\mu_2 b_0^2 \zeta}, \\ C_n &= \tilde{C}_n + c_n = (-b_0 + r_n) e^{-i\mu_2 b_0^2 \zeta}, \end{aligned}$$

where  $p_n$ ,  $q_n$  and  $r_n$  are the small perturbations of the fields in  $n$ -th unit cell.

The linearized system of equation for these perturbations takes the form

$$\begin{aligned} i \frac{\partial p_n}{\partial \zeta} &= -\varrho p_n + (q_n + q_{n-1}) + (r_n + r_{n-1}), \\ i \frac{\partial q_n}{\partial \zeta} &= (p_n + p_{n+1}) + \varrho(q_n + q_n^*), \\ i \frac{\partial r_n}{\partial \zeta} &= (p_n + p_{n+1}) + \varrho(r_n + r_n^*). \end{aligned} \quad (7)$$

Here  $\varrho = \mu_2 b_0^2$ .

Let there be  $N = 2M + 1$  waveguides in RNOWA. The fields in  $n$ -th unit cell are presented as the Fourier

serieses

$$\begin{aligned} p_n &= \sum_{s=-M}^{s=M} \left( p_s e^{2\pi i n s/M} + \bar{p}_s e^{-2\pi i n s/M} \right), \\ q_n &= \sum_{s=-M}^{s=M} \left( q_s e^{2\pi i n s/M} + \bar{q}_s e^{-2\pi i n s/M} \right), \\ r_n &= \sum_{s=-M}^{s=M} \left( r_s e^{2\pi i n s/M} + \bar{r}_s e^{-2\pi i n s/M} \right). \end{aligned} \quad (8)$$

Substitution of the (8) in the equations (7) with taking into account the orthogonality of the harmonic functions results in the following equations for modes

$$\begin{aligned} i \frac{\partial p_s}{\partial \zeta} &= -\varrho p_s + \kappa(s)^*(q_s + r_s), \\ i \frac{\partial \bar{p}_s}{\partial \zeta} &= -\varrho \bar{p}_s + \kappa(s)(\bar{q}_s + \bar{r}_s), \\ i \frac{\partial q_s}{\partial \zeta} &= \kappa(s)p_s + \varrho(q_s + \bar{q}_s^*), \\ i \frac{\partial \bar{q}_s}{\partial \zeta} &= \kappa(s)^*\bar{p}_s + \varrho(\bar{q}_s + q_s^*), \\ i \frac{\partial r_s}{\partial \zeta} &= \kappa(s)p_s + \varrho(r_s + \bar{r}_s^*), \\ i \frac{\partial \bar{r}_s}{\partial \zeta} &= \kappa(s)^*\bar{p}_s + \varrho(\bar{r}_s + r_s^*). \end{aligned}$$

Here

$$\kappa(s) = 2 \cos(\pi s/M) e^{i\pi s/M}.$$

In following the mode marks  $s$  can be omitted, as long as it will be necessary to indicate mode marks. If the new functions  $w = \kappa p$ ,  $\bar{w} = \kappa^* \bar{p}$  are introduced, this system of equations will take the form

$$\begin{aligned} i \frac{\partial w}{\partial \zeta} &= -\varrho w + |\kappa|^2(q + r), & i \frac{\partial \bar{w}}{\partial \zeta} &= -\varrho \bar{w} + |\kappa|^2(\bar{q} + \bar{r}), \\ i \frac{\partial q}{\partial \zeta} &= w + \varrho(q + \bar{q}^*), & i \frac{\partial \bar{q}}{\partial \zeta} &= \bar{w} + \varrho(\bar{q} + q^*), \\ i \frac{\partial r}{\partial \zeta} &= w + \varrho(r + \bar{r}^*), & i \frac{\partial \bar{r}}{\partial \zeta} &= \bar{w} + \varrho(\bar{r} + r^*). \end{aligned} \quad (9)$$

From these equations one can find the closed system of three equations

$$\begin{aligned} -\frac{\partial^2 q}{\partial \zeta^2} &= -\varrho \bar{w}^* + |\kappa|^2(q + r), \\ -\frac{\partial^2 r}{\partial \zeta^2} &= -\varrho \bar{w}^* + |\kappa|^2(q + r), \\ -\frac{\partial^2 \bar{w}^*}{\partial \zeta^2} &= (2|\kappa|^2 + \varrho^2)\bar{w}^* + \varrho|\kappa|^2(q + r). \end{aligned}$$

If the variable  $u = q + r$ ,  $\tilde{u} = q - r$ ,  $\bar{w}^* = v$  are used, the

either system of equation can be written

$$\begin{aligned} \frac{\partial^2 v}{\partial \zeta^2} + (2|\kappa|^2 + \varrho^2)v + \varrho|\kappa|^2 u &= 0, \\ \frac{\partial^2 u}{\partial \zeta^2} - 2\varrho v + 2|\kappa|^2 u &= 0, \\ \frac{\partial^2 \tilde{u}}{\partial \zeta^2} &= 0. \end{aligned} \quad (10)$$

Thus the variable  $\tilde{u} = q - r$  varies as  $\tilde{u} = \tilde{u}_0 + \tilde{u}_1 \zeta$ . Hence, the small perturbations vary proportionally with the distance  $\zeta$ . In this since the flat band solution is unstable. However, it is a weak instability.

If the initial variable  $q, r, v = \bar{w}^*$  are considered, the corresponding characteristic equation takes the form

$$\begin{vmatrix} \lambda^2 + (2|\kappa|^2 + \varrho^2) & \varrho|\kappa|^2 & \varrho|\kappa|^2 \\ -\varrho & \lambda^2 + 2|\kappa|^2 & |\kappa|^2 \\ -\varrho & |\kappa|^2 & \lambda^2 + 2|\kappa|^2 \end{vmatrix} = 0.$$

The calculation of this determinant results in the algebraic equation

$$\lambda^2 [(\lambda^2 + 2|\kappa|^2 + \varrho^2)(\lambda^2 + 2|\kappa|^2) + 2\varrho^2|\kappa|^2] = 0. \quad (11)$$

The roots of equation (11)  $\lambda^2 = 0$  are evidence for the linear increasing of the small perturbations. Another roots can be found from reduced characteristic equation

$$(\lambda^2 + 2|\kappa|^2 + \varrho^2)(\lambda^2 + 2|\kappa|^2) + 2\varrho^2|\kappa|^2 = 0. \quad (12)$$

The changing  $\lambda^2 = 2|\kappa|^2 \xi$  results in equation  $(1 + \xi)(1 + \xi + \mu) + \mu = 0$ , where  $\mu = \varrho^2/(2|\kappa|^2)$ . It follows that roots of this equation read as

$$\xi_{1,2} = -\left(1 + \frac{\mu}{2}\right) \pm \sqrt{D_e},$$

where  $D_e = \mu^2/4 - \mu$ . Thus, the roots of equation (12) are given by the expressions

$$\lambda_{1,2}^{(\pm)} = \pm \sqrt{2|\kappa|} \left[ -\left(1 + \frac{\mu}{2}\right) \pm \sqrt{D_e} \right]^{1/2}. \quad (13)$$

The instability takes place if  $\text{Re}(\lambda_1^{(\pm)}) > 0$  or/and  $\text{Re}(\lambda_2^{(\pm)}) > 0$ .

In a linear case, where  $\mu = 0$ ,  $\lambda_{1,2}^{(\pm)} = \pm 2i|\kappa|$ . It means that there is no the exponential increasing of the perturbations. However, any small perturbations lead to spreading the electromagnetic waves in transversal direction. It is due to  $\lambda^2 = 0$ . Thus the discrete diffraction in a linear 1D rhombic waveguide array takes place [20].

The roots of the equations (12) can be written as

$$\lambda_1^{(\pm)} = \pm \sqrt{2|\kappa|} \sqrt{\xi_1}, \quad \lambda_2^{(\pm)} = \pm \sqrt{2|\kappa|} \sqrt{\xi_2}.$$

If  $0 < \mu < 4$  the discriminant  $D_e$  is negative one,  $D_e = -\mu(4 - \mu)/4$ , hence

$$\xi_{1,2} = -\left(1 + \frac{\mu}{2}\right) \pm i\sqrt{|D_e|}.$$

By extracting square root from  $\xi_{1,2}$  one can obtain the expression for roots of the equation (12):

$$\begin{aligned}\lambda_1^{(\pm)} &= \Lambda(\cosh \phi^+ + i \sinh \phi^+), \\ \lambda_2^{(\pm)} &= \Lambda(\cosh \phi^- + i \sinh \phi^-),\end{aligned}$$

where

$$\Lambda = \sqrt{2}|\kappa| \left(1 + \frac{\mu}{2}\right)^{1/2}, \quad \sinh 2\phi = \pm \frac{\sqrt{|D_e|}}{1 + \mu/2}.$$

As  $\text{Re}(\lambda_1^{(\pm)}) > 0$  and  $\text{Re}(\lambda_2^{(\pm)}) < 0$ , the flat band solution under consideration is unstable in the region  $0 < \mu < 4$ .

If  $\mu > 4$  the discriminant  $D_e = \mu(\mu - 4)/4$  is positive. Hence  $\xi_{1,2}$  is real value and

$$\xi_1 = -\left(1 + \frac{\mu}{2}\right) + \sqrt{D_e}, \quad \xi_2 = -\left(1 + \frac{\mu}{2}\right) - \sqrt{D_e}.$$

For  $\xi_1$  the following expression

$$\xi_1 = -\left(1 + \frac{\mu}{2}\right) + \frac{\mu}{2} \sqrt{1 - \frac{4}{\mu^2}} = -1 - \frac{\mu}{2} \left(1 - \sqrt{1 - \frac{4}{\mu^2}}\right),$$

can be found. It is negative at  $\mu > 4$ . From the definition  $\xi_2$  it follows that  $\xi_2 < 0$ . Thus,  $\text{Re}(\lambda_1^{(\pm)}) = 0$ ,  $\text{Re}(\lambda_2^{(\pm)}) = 0$ . Hence, the flat band solution is stable in the region  $\mu > \mu_c = 4$ .

So, the flat band solution (6) is unstable if the radiation power in waveguide is less than some threshold power. This solution will be stable if the power is more than threshold value.

Using the definition of the relevant parameters

$$\varrho = \mu_2 b_0^2, \quad |\kappa(s)| = 2|\cos(\pi s/M)|,$$

one can write the stability condition in the form

$$\mu_2 b_0^2 \geq 2\sqrt{2}|\cos(\pi s/M)|. \quad (14)$$

If the normalized power per mode  $b_0$  will be greater than the critical value ( $b_{0c}^2 = 2\sqrt{2}\mu_2^{-1}|\cos(\pi s/M)|$ ) the perturbations will not increase exponentially. It should be mentioned that the critical value  $b_{0c}$  is depended on mode marker  $s$ . Hence only part of the modes having markers, which are belong to the interval

$$\frac{\pi}{2} > \frac{\pi s}{M} \geq \arccos \frac{\mu_2 b_0^2}{2\sqrt{2}},$$

will be stable. All modes of the flat band will be stable if the condition  $\mu_2 b_0^2 \geq 1$  is held.

## V. CONCLUSION

The rhombic nonlinear optical waveguide array is considered in the paper. In the case of linear waveguides the waveguide array of this kind has been investigated in [14, 17, 18]. It was shown that all (normal) mode of this waveguide array are separated on three groups or bands in 1D space of the wave vectors. The two bands are populated by the modes describing the discrete diffraction in waveguide array. Third band contains the modes that describes the wave propagating without diffraction. This band was named as the flat band.

In the RNOWA the flat band analog exists. There is the solutions of the system of equation of the RNOWA describing the diffractionless waves propagation. However, both the linear and nonlinear the flat band solution are weak instable. The small perturbations increase directly with the first power of distance along the waveguide. In the case of nonlinear waveguides the small perturbations grow exponentially. But with power increasing the part of modes began be stable. The all modes became stable if the power of radiation in waveguide is greater then the threshold value. Theses flat band solution are stable but not asymptotic stable. The phenomenon is similar to the self-focusing in a nonlinear bulk medium.

As it was pointed above, there are two kind of the electric field distributions: (a)  $A_n = 0$ ,  $B_n = -C_n$  and (b)  $A_n = (-1)^n A$ ,  $B_n = (-1)^n B$ ,  $C_n = (-1)^{n+1} C$ , for which the power flux between unit cells of the RNOWA is equal to zero. Here the case of (a) was investigated. As for the second case, there is discrete analogy of the modulation instability for a cubic nonlinear bulk material. There is threshold power for each mode and the threshold power for all band. If the power of radiation in waveguide of RNOWA is greater than this threshold the perturbations are increased exponentially with distance.

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- [1] V. Yannopapas *Int. J. Mod. Phys. B.* **28**, 1441005 [15 pages] (2014).  
[2] T. Ochiai, M. Onoda *Phys. Rev. B.* **80** 155103 (2009).  
[3] D. Dragoman *J. Opt.* **16**, 015710 (2014).  
[4] Cheng He, Liang Lin, Xiao-Chen Sun, Xiao-Ping Liu, Ming-Hui Lu, Yan-Feng Chen. *Int. J. Mod. Phys. B.* **28**,

- 1441001 [15 pages] (2014).  
[5] S. Longhi *Laser & Photonics Reviews.* **3**, 243–261 (2009).  
[6] B. M. Rodri'guez-Lara, H. M. Moya-Cessa. *Phys. Rev. A.* **89**, 015803 (2014)  
[7] A. Greenleaf, Ya. Kurylev, M. Lassas, G. Uhlmann. *Phys. Rev. Lett.* **99**, 183901 (2007).

- [8] E.E. Narimanov and A.V. Kildishev, *Appl. Phys. Lett.* **95**, 041106 (2009).
- [9] I.I. Smolyaninov, E. Hwang, and E.E. Narimanov, *Phys. Rev. B.* **85**, 235122 (2012).
- [10] I.I. Smolyaninov, *Opt. Express* **21**, 14918 (2013).
- [11] D. Guzman-Silva, C. Mejia-Cortes, M.A. Bandres, M.C. Rechtsman, S. Weimann, S. Nolte, M. Segev, A. Szameit, and R.A. Vicencio, *New J. Phys.* **16**, 063061 (2014).
- [12] Yun-tuan Fang, Han-Qing He, Jian-xia Hu, Lin-kun Chen, and Zhang Wen. *Phys. Rev. A.* **91**, 033827 (2015).
- [13] R.A. Vicencio, C. Cantillano, L. Morales-Inostroza, B. Real, C. Mejia-Cortes, St. Weimann, Al. Szameit, and M.I. Molina. *Phys.Rev.Lett.* **114**, 245503 (2015).
- [14] St. Longhi, *Opt. Lett.* **39**, 5892–5895 (2014).
- [15] A. Maimistov, *J.Phys. Conference Series* **613**, 012011 (2015)
- [16] A.I. Maimistov, I.R. Gabitov, *J.Phys. Conference Series* **714**, 012013 (4 pp) (2016).
- [17] S. Mukherjee and R.R. Thomson. *Opt. Lett.* **40**(23), 5443–5446 (2015).
- [18] S. Mukherjee, A. Spracklen, D. Choudhury, N. Goldman, P. Ohberg, E. Andersson, and R.R. Thomson, *Phys.Rev.Lett.* **114**, 245504 (2015).
- [19] Yuanyuan Zong, Shiqiang Xia, Liqin Tang, Daohong Song, Yi Hu, Yumiao Pei, Jing Su, Yigang Li, and Zhi-gang Chen. *Opt.Express* **24**, 8877–8885 (2016).
- [20] A.I. Maimistov and V.A. Patrikeev, Electromagnetic Wave Propagation in a Quasi-1D Rhombic Linear Optical Waveguide Array, Preprint arXiv:1606.03697 [physics.optics].